

Diamagnetism versus Paramagnetism in charged spin-1 Bose gases

Xiaoling Jian, Jihong Qin, and Qiang Gu*

Department of Physics, University of Science and Technology Beijing, Beijing 100083, China

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It has been suggested that either diamagnetism or paramagnetism of Bose gases, due to the charge or spin degrees of freedom respectively, appears solely to be extraordinarily strong. We investigate magnetic properties of charged spin-1 Bose gases in external magnetic field, focusing on the competition between the diamagnetism and paramagnetism, using the Lande-factor g of particles to evaluate the strength of paramagnetic effect. We propose that a gas with $g < 1/\sqrt{8}$ exhibits diamagnetism at all temperatures, while a gas with $g > 1/2$ always exhibits paramagnetism. Moreover, a gas with the Lande-factor in between shows a shift from paramagnetism to diamagnetism as the temperature decreases. The paramagnetic and diamagnetic contributions to the total magnetization density are also calculated in order to demonstrate some details of the competition.

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I. INTRODUCTION

The Bose gas plays a significant role in understanding a series of exotic quantum phenomena, including superfluidity and superconductivity. It is well known that the ideal Bose gas exhibits the Bose-Einstein condensation (BEC) below a critical temperature. In 1938, London first connected BEC to the λ transition in Helium.¹ In 1946, Ogg proposed that the BEC of bosonic electron-pairs might result in superconductivity.² Furthermore, Schafroth³ and Blatt and Butler⁴ showed that an ideal gas of charged bosons exhibits the essential equilibrium features of a superconductor. Although the BCS theory⁵ revealed that electrons in a superconductor form Cooper-pairs, as opposed to real-space pairs, the Schafroth-Blatt-Butler theory is helpful to the understanding of superconductivity and more importantly it stimulated research interest in charged Bose gases.

The charged Bose gas (CBG) is solely of academic interest in its own right. Especially, it exhibits nontrivial magnetic properties. For example, the condensation phenomenon in CBGs is strongly affected by the external magnetic field, owing to the quantization of the orbital motion of charged particles in magnetic field.^{3,6-12} Schafroth clearly indicated that an arbitrarily small value of the magnetic field introduces qualitative changes: BEC does no longer occur.³ May extended this idea to a d -dimensional CBG and found that it can condense only for $d > 4$.⁷ This point was reexamined by other researchers based on different methods.^{10,11} Meanwhile, the orbital motion results in extremely large Landau diamagnetism in CBGs. It was pointed out that the 3-dimensional CBG displays Meissner effect at low temperatures.^{3,12,13} More recently, Alexandrov quantitatively accounted for the enhanced normal-state diamagnetism of superconducting cuprates using the CBG model.¹⁴

On the other hand, neutral bosons with spin (spinor bosons) have also been studied theoretically.¹⁵ The realization of spinor Bose condensate in optically trapped alkali atoms¹⁶ stimulates new research interest. The constituent atoms, such as ⁸⁷Rb, ²³Na, and ⁷Li have (hy-

perfine) spin degrees of freedom and thus a magnetic moment. Their spin degrees of freedom become active in purely optical traps and thus investigation of their magnetic properties becomes possible. Yamada¹⁵, and Simkin and Cohen¹⁷ calculated the magnetization of neutral spinor bosons in magnetic field and found that once BEC takes place, the magnetization remains finite even if $H = 0$, as if the system was magnetized spontaneously. The zero-field susceptibility tends to diverge as the temperature goes down to the BEC temperature. Moreover, rigorous proofs presented by Eisenberg and Lieb show that the magnetization and zero-field susceptibility at finite temperatures exceed that of a pure paramagnet.¹⁸ All the results indicate that the neutral spinor bosons take on extraordinary paramagnetic effect in magnetic field.

Bearing in mind the two contrary features of the Bose gas, one immediate question which must be raised is: which kind of magnetism will manifest itself if the bosons possess both the charge and spin degrees of freedom? Analogously, this question has been answered for charged Fermi gases, e.g., the electron gas, where both the paramagnetism and diamagnetism are relatively weak. For the electron gas, the diamagnetic part of the zero-field susceptibility is one-third (in absolute value) of the paramagnetic part, so altogether the gas is paramagnetic. For charged spinor Bose gas, since both the paramagnetism and diamagnetism can be extremely large, their competition is a more interesting problem.

In this paper, we study the competition between paramagnetism and diamagnetism of a charged spin-1 Bose gas in external magnetic field. In Section II, a model consisting of both the Landau diamagnetic effect and Pauli paramagnetic effect is proposed. The total magnetization density as well as its paramagnetic and diamagnetic parts are calculated respectively. Section III presents a detailed discussion of the obtained results. In Section IV, a brief summary is given.

II. THE MODEL

The orbital motion of a charged boson with charge q and mass m^* in a constant magnetic field B is quantized into the Landau levels,

$$\epsilon_{jk_z}^l = \left(\frac{1}{2} + j\right)\hbar\omega + \frac{\hbar^2 k_z^2}{2m^*}, \quad (1)$$

where $j = 0, 1, 2, \dots$ labels different Landau levels and $\omega = qB/(m^*c)$ is the gyrofrequency. Since $\omega \propto B$, ω can be used to indicate the magnitude of the magnetic field in the following discussions. We assume the magnetic field is in the z direction. Each Landau level is degenerate with degeneracy equal to

$$D_L = \frac{qBL_x L_y}{2\pi\hbar c}. \quad (2)$$

Here we suppose the gas is in a box with $L_i \rightarrow \infty$, where L_i is the length of the box in the i th direction. For a spin-1 boson, the Zeeman energy levels split in the magnetic field due to the intrinsic magnetic moment associated with the spin degree of freedom,

$$\epsilon_\sigma^{ze} = -g \frac{\hbar q}{m^* c} \sigma B, \quad (3)$$

where σ refers to the spin- z index of Zeeman state $|F=1, m_F=\sigma\rangle$ ($\sigma = +1, 0, -1$) and g is the Lande-factor. The quantization of the orbital motion and the Zeeman effect give rise to the Landau diamagnetism and Pauli paramagnetism, respectively.

We consider an assembly of N bosons, whose effective Hamiltonian reads

$$\bar{H} - \mu N = D_L \sum_{j,k_z,\sigma} (\epsilon_{jk_z}^l + \epsilon_\sigma^{ze} - \mu) n_{jk_z\sigma}, \quad (4)$$

where μ is the chemical potential. The charged spinor bosons have been discussed theoretically in the context of relativistic pair creation¹⁹. However, magnetism of charged spinor Bose gases is less studied and of major interest in the present work.

The grand thermodynamic potential is formally expressed as

$$\begin{aligned} \Omega_{T \neq 0} &= -\frac{1}{\beta} \ln \text{Tr} e^{-\beta(\bar{H} - \mu N)} \\ &= \frac{1}{\beta} D_L \sum_{j,k_z,\sigma} \ln[1 - e^{-\beta(\epsilon_{jk_z}^l + \epsilon_\sigma^{ze} - \mu)}], \end{aligned} \quad (5)$$

where $\beta = (k_B T)^{-1}$. Converting the sum over k_z to continuum integral, we get

$$\begin{aligned} \Omega_{T \neq 0} &= \frac{\omega m^* V}{(2\pi)^2 \hbar \beta} \sum_{j=0}^{\infty} \sum_{\sigma} \int dk_z \\ &\times \ln\{1 - e^{-\beta[(j+\frac{1}{2})\hbar\omega + \frac{\hbar^2 k_z^2}{2m^*} - g \frac{\hbar q}{m^* c} \sigma B - \mu]}\}, \end{aligned} \quad (6)$$

where V is the volume of the system. Then using the Taylor expansion and performing the integral over k_z , we have

$$\begin{aligned} \Omega_{T \neq 0} &= -\frac{\omega V}{\hbar^2} \left(\frac{m^*}{2\pi\beta}\right)^{3/2} \\ &\times \sum_{l=1}^{\infty} \sum_{\sigma} \frac{l^{-\frac{3}{2}} e^{-l\beta(\frac{\hbar\omega}{2} - g \frac{\hbar q}{m^* c} \sigma B - \mu)}}{1 - e^{-l\beta\hbar\omega}}. \end{aligned} \quad (7)$$

This treatment has been used by Standen and Toms¹¹ to deal with scalar Bose gases. As they mentioned, this theory is more reliable at high temperature. Similarly, we introduce some compact notation for the class of sums,

$$\Sigma_\kappa[\alpha, \delta] = \sum_{l=1}^{\infty} \frac{l^{\alpha/2} e^{-l\kappa(\alpha+\delta)}}{(1 - e^{-l\kappa})^\kappa}, \quad (8)$$

where $x = \beta\hbar\omega$ and $\mu + g\hbar q\sigma B/(m^*c) = (\frac{1}{2} - \varepsilon)\hbar\omega$. With this notation we may rewrite Eq. (7) as

$$\Omega_{T \neq 0} = -\frac{\omega V}{\hbar^2} \left(\frac{m^*}{2\pi\beta}\right)^{3/2} \sum_{\sigma} \Sigma_1[-D, 0], \quad (9)$$

where $D = 3$. Then the density of particles $n = N/V$ can be derived from the thermodynamic potential,

$$\begin{aligned} n &= -\frac{1}{V} \left(\frac{\partial \Omega}{\partial \mu}\right)_{T,V} \\ &= x \left(\frac{m^*}{2\pi\beta\hbar^2}\right)^{3/2} \sum_{\sigma} \Sigma_1[2 - D, 0]. \end{aligned} \quad (10)$$

The magnetization density M is written as

$$\begin{aligned} M_{T \neq 0} &= -\frac{1}{V} \left(\frac{\partial \Omega}{\partial B}\right)_{T,V} \\ &= \frac{\hbar q}{m^* c} \left(\frac{m^*}{2\pi\beta\hbar^2}\right)^{3/2} \sum_{\sigma} \left\{ \Sigma_1[-D, 0] \right. \\ &\quad \left. + x(g\sigma - \frac{1}{2})\Sigma_1[2 - D, 0] - x\Sigma_2[2 - D, 1] \right\}. \end{aligned} \quad (11)$$

It is convenient to introduce some dimensionless parameters, such as $\bar{M} = m^* c M / (n \hbar q)$, $\bar{\omega} = \hbar\omega / (k_B T^*)$, $t = T/T^*$ and $x' = \bar{\omega}/t$, to re-express equations (10) and (11),

$$1 = \bar{\omega} t^{1/2} \sum_{\sigma} \Sigma'_1[2 - D, 0], \quad (12)$$

$$\begin{aligned} \bar{M}_{T \neq 0} &= t^{3/2} \sum_{\sigma} \left\{ \Sigma'_1[-D, 0] + x'(g\sigma - \frac{1}{2}) \right. \\ &\quad \left. \times \Sigma'_1[2 - D, 0] - x'\Sigma'_2[2 - D, 1] \right\}. \end{aligned} \quad (13)$$

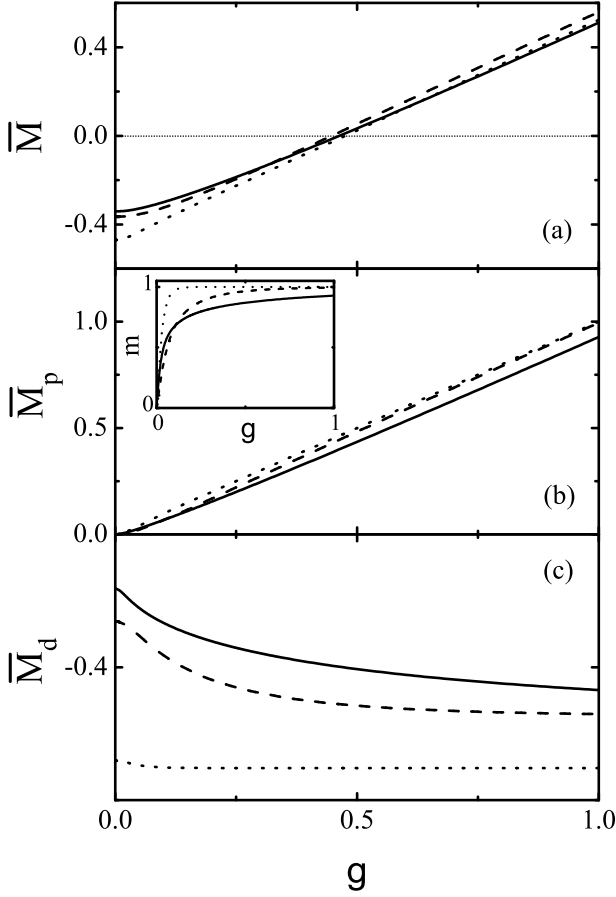


FIG. 1: (a) The total magnetization density (\overline{M}), (b) the paramagnetization density (\overline{M}_p), and (c) the diamagnetization density (\overline{M}_d) as a function of g for fixed magnetic fields at $t = 0.1$. Here the solid line, dashed line and dotted line correspond to $\overline{\omega} = 0.05, 0.3$, and 3 , respectively. Inset: m as a function of g .

Here the characteristic temperature T^* is given by $k_B T^* = 2\pi\hbar^2 n^{2/3}/m^*$. The Bose-Einstein condensation temperature of spin-1 Bose gas with density n is just defined as $k_B T_c = 2\pi\hbar^2 n^{2/3}/\{m^*[3\zeta(3/2)]^{2/3}\} \approx k_B T^*/3.945$. The dimensionless notation $\Sigma'_\kappa[\alpha, \delta]$ should be

$$\Sigma'_\kappa[\alpha, \delta] = \sum_{l=1}^{\infty} \frac{l^{\alpha/2} e^{-lx'(\varepsilon+\delta)}}{(1 - e^{-lx'})^\kappa}, \quad (14)$$

where $\mu' = \mu/(k_B T^*)$ and $\mu' + g\sigma\overline{\omega} = (\frac{1}{2} - \varepsilon)\overline{\omega}$. The two variables μ' and \overline{M} are determined by Eqs. (12) and (13).

III. RESULTS AND DISCUSSIONS

In our model, both the charge and spin degrees of freedom are taken into account, which are described respectively by the Landau energy term $\epsilon_{jk_z}^l$ and the Zeeman

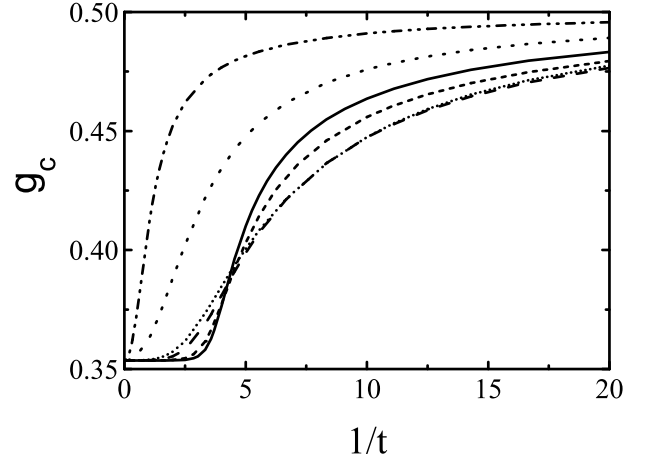


FIG. 2: Plots of the critical value of Lande-factor, g_c as a function of $1/t$ for fixed values of $\overline{\omega}$. The field is chosen as: $\overline{\omega} = 10$ (dash-dot-dotted line), 3 (dotted line), 0.5 (short dotted line), 0.3 (dashed line), 0.1 (short-dashed line), and 0.05 (solid line).

energy term ϵ_σ^{ze} in the Hamiltonian (4). In the case that the Lande-factor g tends to zero, the model degenerates into a charged scalar boson model which exhibits strong diamagnetism as already intensively discussed.⁷⁻¹² As g becomes larger, the paramagnetic effect is strengthened.

We calculate the dimensionless magnetization density \overline{M} as a function of g , as shown in Fig. 1(a). \overline{M} is negative in the small g region, which means that the diamagnetism dominates. The absolute value of \overline{M} is larger in the stronger field $\overline{\omega}$. For each given value of $\overline{\omega}$, \overline{M} increases monotonically with g . \overline{M} changes its sign from negative to positive at a critical value of g , noted as g_c hereinafter, reflecting that the paramagnetism becomes dominant as g increases. Note that the slope of the \overline{M} curve is dependent on g . When g is near to zero, \overline{M} increases slowly but the slope rises quickly with g . It means that the interplay between diamagnetism and paramagnetism is complex and nonlinear. However, in the strong paramagnetic region, \overline{M} grows almost linearly with g .

Figure 1(b) plots the paramagnetic contribution to \overline{M} (named as the paramagnetization density), $\overline{M}_p = gm$, with an inset showing $m = n_1 - n_{-1}$, and Figure 1(c) shows the diamagnetic contribution to \overline{M} (named as the diamagnetization density), $\overline{M}_d = \overline{M} - \overline{M}_p$. The increasing tendency of \overline{M}_p is similar to that of \overline{M} . It is noteworthy that the diamagnetization density is not suppressed, but enhanced as g becomes larger. Comparing Figs 1(a), 1(b) and 1(c), it can be seen that the increase in \overline{M} comes from the paramagnetic effect. In the small g region, both \overline{M}_p and \overline{M}_d are strengthened nonlinearly with increasing g . As shown in the inset of Fig. 1(b), m grows very quickly. Nevertheless, both m and \overline{M}_d flatten out in the large g region. So the slope of \overline{M} curve is mainly due to the paramagnetization.

According to discussions above, the critical value of the Lande-factor, g_c , is an important parameter to describe

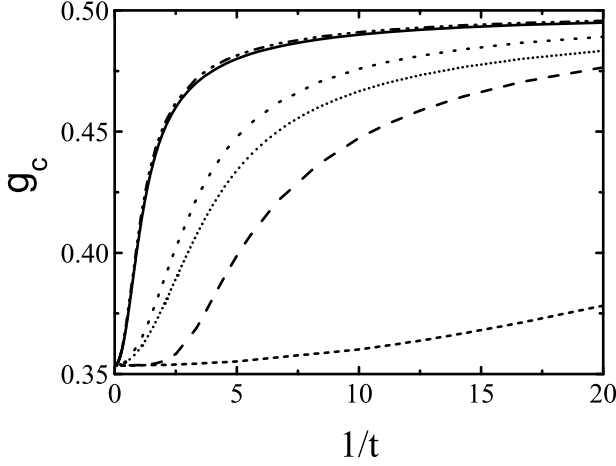


FIG. 3: Curves of $g_c - 1/t$ obtained respectively from the Bose-Einstein (BE) and Maxwell-Boltzmann (MB) statistics with fixed values of $\bar{\omega}$. Where $\bar{\omega} = 10$ (dash-dot-dotted line, BE; solid line, MB), 3 (dotted line, BE; short dotted line, MB), and 0.3 (dashed line, BE; short dashed line, MB).

the competition between the diamagnetism and paramagnetism. g_c is a function of the temperature t and the magnetic field $\bar{\omega}$. Figure 2 shows g_c as a function of $1/t$ in different magnetic fields $\bar{\omega} = 10, 3, 0.5, 0.3, 0.1$ and 0.05 . Obviously, g_c varies monotonically with the temperature t , while its dependence on the field $\bar{\omega}$ is not simple. For example, in the low temperature region, g_c decreases with decreasing $\bar{\omega}$ at a given temperature as $\bar{\omega}$ is still larger than 0.3, then it rises up as $\bar{\omega}$ goes down further from $\bar{\omega} \approx 0.3$.

As already mentioned, the results of our theory are more credible at high temperature. In the high temperature limit, g_c seems universal with respect to different choices of magnetic field, $g_c|_{t \rightarrow \infty} \approx 0.35356$. For a given magnetic field, g_c increases as the temperature falls down. This suggests that the diamagnetic region is larger at low temperatures than at high temperatures. Although the exact value of g_c can not be obtained at very low temperatures, its variation trend can be estimated from Fig. 2. It seems that g_c ranges from 0.475 to 0.50 in various magnetic fields.

It is useful to reexamine the high temperature behaviors of g_c by generalizing above calculations to a spin-1 Boltzmann gas, since the Bose-Einstein statistics reduces to Maxwell-Boltzmann statistics in the high temperature limit. The grand thermodynamic potential based on the Maxwell-Boltzmann statistics reads

$$\Omega_{T \neq 0} = -\frac{1}{\beta} \sum_{j, k_z, \sigma} D_L e^{-\beta(\epsilon_{j k_z}^l + \epsilon_{\sigma}^z e - \mu)}. \quad (15)$$

Then equations of the dimensionless chemical potential μ' and magnetization density \bar{M}^B are derived respectively,

$$1 = \bar{\omega} t^{1/2} \sum_{\sigma} \frac{e^{-\frac{1}{t}(\frac{\bar{\omega}}{2} - g\sigma\bar{\omega} - \mu')}}{1 - e^{-\frac{\bar{\omega}}{t}}} \quad (16)$$

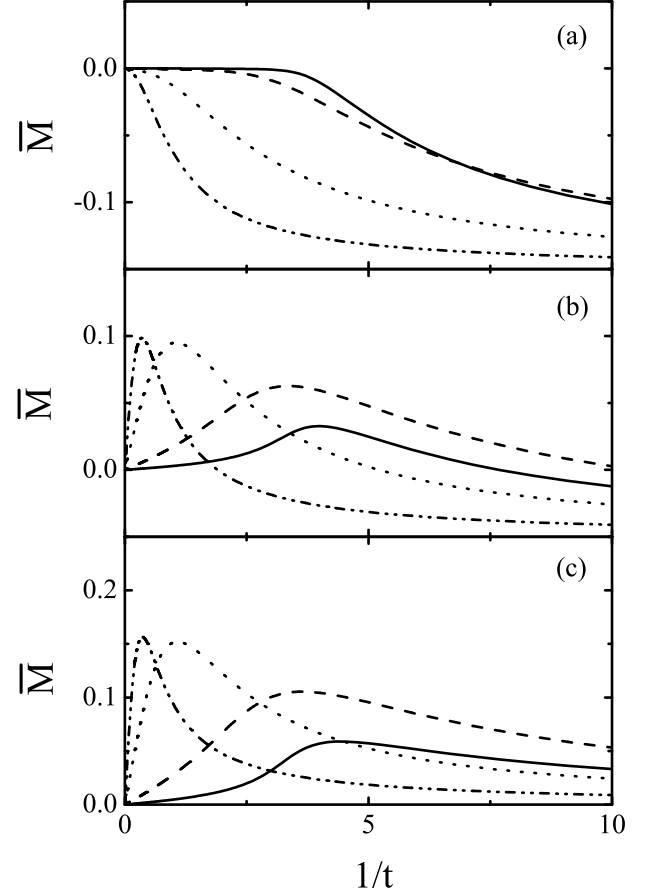


FIG. 4: Shown are plots of dimensionless \bar{M} as a function of $1/t$ for each given value of $\bar{\omega}$ at a fixed g . The value of $\bar{\omega}$ were in sequence as 10 (dash-dot-dotted line), 3 (dotted line), 0.3 (dashed line), and 0.05 (solid line). (a) corresponds to $g = 0.35$. (b) corresponds to $g = 0.45$. (c) corresponds to $g = 0.5$.

and

$$\bar{M}_{T \neq 0}^B = t^{3/2} \sum_{\sigma} \left\{ \frac{e^{-\frac{1}{t}(\frac{\bar{\omega}}{2} - g\sigma\bar{\omega} - \mu')}}{1 - e^{-\frac{\bar{\omega}}{t}}} \times \left[1 + \frac{\bar{\omega}}{t} \left(g\sigma - \frac{1}{2} - \frac{e^{-\frac{\bar{\omega}}{t}}}{1 - e^{-\frac{\bar{\omega}}{t}}} \right) \right] \right\}. \quad (17)$$

Substituting Eq. (16) into Eq. (17), yields

$$\bar{M}_{T \neq 0}^B = \frac{1}{x'} - \frac{1}{2} - \frac{1}{e^{x'} - 1} + \frac{g(e^{2gx'} - 1)}{e^{2gx'} + e^{gx'} + 1}. \quad (18)$$

An analytical formula for g_c can be obtained,

$$\frac{1}{2} = \frac{1}{x'} - \frac{1}{e^{x'} - 1} + \frac{g_c(e^{2g_c x'} - 1)}{e^{2g_c x'} + e^{g_c x'} + 1}. \quad (19)$$

The value of g_c can be derived from Eq. (19) exactly in two limit cases: $g_c|_{t \rightarrow \infty} = 1/\sqrt{8} \approx 0.35355$ and $g_c|_{t \rightarrow 0} = 1/2$. The value of g_c for a Boltzmann gas is reasonably equal to that of a Bose gas in the high temperature limit. Figure 3 plots the numerical solutions of

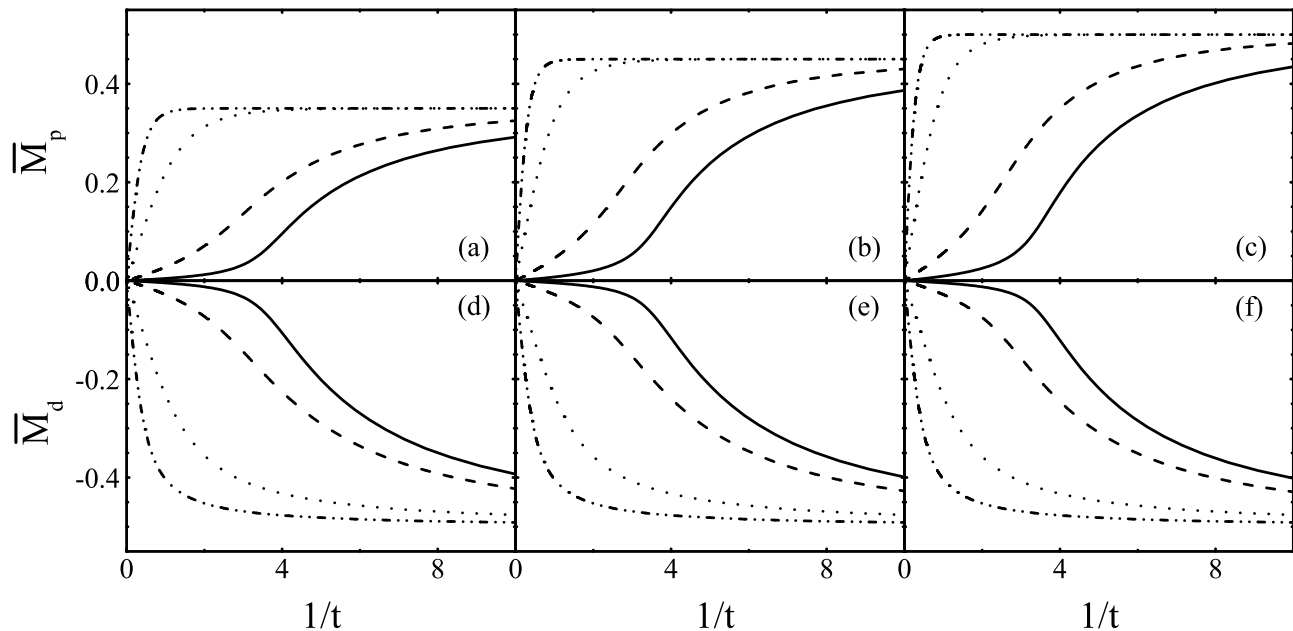


FIG. 5: The dimensionless \overline{M}_p and \overline{M}_d as a function of $1/t$ for fixed values of $\overline{\omega}$. From left to right $g = 0.35, 0.45$ and 0.5 , respectively. For each given value of g , the field is chosen as: $\overline{\omega} = 10$ (dash-dot-dotted line), 3 (dotted line), 0.3 (dashed line), and 0.05 (solid line), respectively.

Eq. (19) and compares with the Bose gas. An interesting point is that the low-temperature-limit value of g_c is also consistent with that of a Bose gas at low temperature but in high magnetic field, although the Maxwell-Boltzmann statistics is just valid in the high temperature region. The stronger the field, the better the accordance.

The $g_c - 1/t$ curves in Figs. 2 and 3 mark the boundary between the diamagnetic and paramagnetic regions. The gas exhibits diamagnetism at all temperatures and in all magnetic field when $g < g_c|_{t \rightarrow \infty} \approx 0.35355$, while always paramagnetism when $g \geq 0.5$. Whereas, the magnetic properties seem more complicated in the intermediate region, $g_c|_{t \rightarrow \infty} < g < 0.5$. Figures 4(a-c) illustrate the dimensionless magnetization density \overline{M} for the three different cases, respectively.

Figure 4(a) denotes the case of $g = 0.35 < g_c|_{t \rightarrow \infty}$. The temperature-dependence of \overline{M} is very similar to that of charged scalar Bose gases,¹¹ as if the paramagnetic effect associated with the spin degree of freedom is thoroughly hidden. The diamagnetism is even stronger at lower temperatures. As the external field tends to be weak, a sharp bend appears gradually on the curve, which located at the point corresponding to the BEC temperature in zero field.¹¹ In our model, the BEC temperature for a spin-1 gas is $(1/t)_c \approx 3.945$. Figure 4(c) shows \overline{M} for a paramagnetic case to the contrary when $g = 0.5$. \overline{M} is always positive in the field at all temperatures. An interesting phenomenon is that the $\overline{M} - 1/t$ curve shows up a peak in this case. The decline in \overline{M} at low temperatures is attributed to the diamagnetic effect. When weakening the magnetic field, the peak is lowered and moves to low temperatures. Fig. 4(b) depicts the case

with g in the intermediate region, which looks quite similar to Fig. 4(c). The key difference is that \overline{M} can change its sign from positive to negative as the temperature decreases, indicating that the system undergoes a shift from paramagnetism to diamagnetism.

Figures 4 demonstrate the total magnetic performance of the charged spin-1 Bose gas. We now turn to examine the underlying paramagnetic and diamagnetic effects for each corresponding case. As shown in Figs. 5, both the paramagnetization density and the diamagnetization density become more stronger at lower temperatures. As the external field is reduced, the strengthening of \overline{M}_p and \overline{M}_d becomes fast near the temperature close to the BEC point in zero field. As g grows, \overline{M}_d is only slightly strengthened but \overline{M}_p increases significantly and thus it can go beyond \overline{M}_d . \overline{M}_p thoroughly exceeds \overline{M}_d when g rises to 0.5 . If g increases further, the paramagnetic effect can be so strong as to cover up the diamagnetic effect completely. Then the total magnetization density \overline{M} becomes monotonously increasing with lowering the temperature and finally reaches a plateau, instead of a peak, at low temperatures.

IV. SUMMARY

This paper has studied the interplay between paramagnetism and diamagnetism of the ideal charged spin-1 Bose gas. The Lande-factor g is introduced to describe the strength of paramagnetic effect caused by the spin degree of freedom. The gas exhibits a shift from diamagnetism to paramagnetism as g increases. The critical

value of g , g_c , is determined by evaluating the dimensionless magnetization density \bar{M} . Our results show that g_c increases monotonically as t decreases. In the high temperature limit, g_c goes to a universal value in all different magnetic fields, $g_c|_{t \rightarrow \infty} = 1/\sqrt{8}$. At low temperatures, our results indicate that g_c ranges from 0.475 to 0.50 as the magnetic field varies. Therefore, a gas with $g < 1/\sqrt{8}$ exhibits diamagnetism at all temperatures, but a gas with $g > 1/2$ always exhibits paramagnetism. In cases where $1/\sqrt{8} < g < 1/2$, the Bose gas undergoes a shift from paramagnetism to diamagnetism as the temperature decreases.

In order to depict some details of the competition between paramagnetism and diamagnetism, the paramagnetic and diamagnetic contributions to the total magnetization density are also calculated. No doubt that the paramagnetism is enhanced with increasing g . Surprisingly, the diamagnetism is not suppressed, but slightly strengthen. This implies that the competition between para- and dia-magnetism is nontrivial. When g is fixed, both the paramagnetism and diamagnetism become stronger as t decreases. As in the scalar case, there is no Bose-Einstein condensation when the magnetic field is present, no matter how small it is. However, evidence of the condensation can be seen in the magnetization density as the magnetic field is reduced.

At last, we briefly discuss experimental aspects possi-

bly relevant to the present work. Although the charged spinor Bose gas has not been realized so far, the up-to-date achievement in experiments makes it attainable perhaps in the near future.²⁰ For example, it is already possible to create ultracold plasmas by photoionization of laser-cooled neutron atoms.²¹ The ions can be regarded as charged bosons if their spin is an integer. The Lande-factor for different magnetic ions could be different. As reported by Killian *et al.*²¹, the temperatures of electrons and ions are as low as 100 mK and 10 μ K, respectively. One can expect that the temperature could be lowered near to the BEC temperature with new advancement in experimental techniques. The diamagnetism-paramagnetism effects manifest themselves near the BEC point. On the other hand, several ferromagnetic superconductors have been discovered since 2000.²² Cooper pairs in such materials are likely in the spin-triplet state, thus behave somewhat like charged spin-1 bosons. Note that the charged spin-1 boson model can not be applied directly to describe triplet superconductors, but it could offer some help in understanding magnetic properties of such materials.

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- * Corresponding author: qgu@ustb.edu.cn
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